

TIMING THE NEXT FOREST INVENTORY

**INCORPORATING FOREST OWNER
PREFERENCES AND FOREST DATA QUALITY**

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PROBLEM OVERVIEW

- In addition to developing a management plan, the forest owner also wants to decide when to **re-inventory** the forest
- The issue is primarily about data quality
 - the quality **degrades** over time, so one way to look at it is as an optimization problem:
 - Conduct a new inventory when **improvements** to the objective function are **maximized**
 - (If an inventory was conducted later, it will produce a lower objective function value)



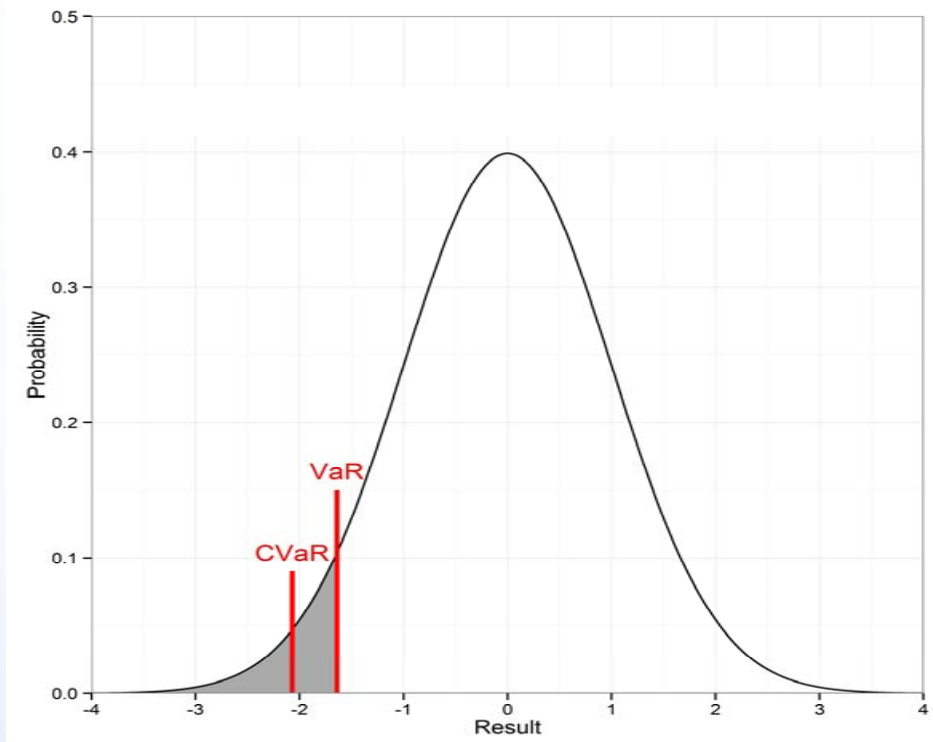
PROBLEM OVERVIEW

- A theoretical case study based on real data will be used as an example:
- 47.3 ha, mainly Scots pine and Norway spruce (some birch)
 - Fairly even age class distribution.
- The forest owner wants to **maximize net present value** while minimizing deviations from even-flow style constraints (measured as the **conditional value at risk**).
- The planning horizon is 30 years, six 5-year periods.
- Uncertainty sources: inventory error (+-10% standard deviation – Basal Area and height), growth model error an AR(1) model (Pietilä et al. 2010).



WHAT IS CVaR:

- CVaR is a measure of risk, evaluated as the average loss exceeding a specific probability.
- Directly related to the Value at Risk (VaR) – which is the expected loss at a specific probability.



HOW TO EVALUATE CVaR:

- CVaR can be evaluated in a stochastic programming framework. Rockafellar and Uryasev (2000) have shown how to utilize it in an optimization framework. CVaR can be used as a constraint or as a part of the objective function.

$$CVaR = Z + \frac{1}{(1 - \alpha)N} \sum_{n=1}^N [L_n - Z]^+$$

- where:

Z - is the VaR (a decision variable) L_n - Loss of scenario n

α - is the probability N - total number of scenarios



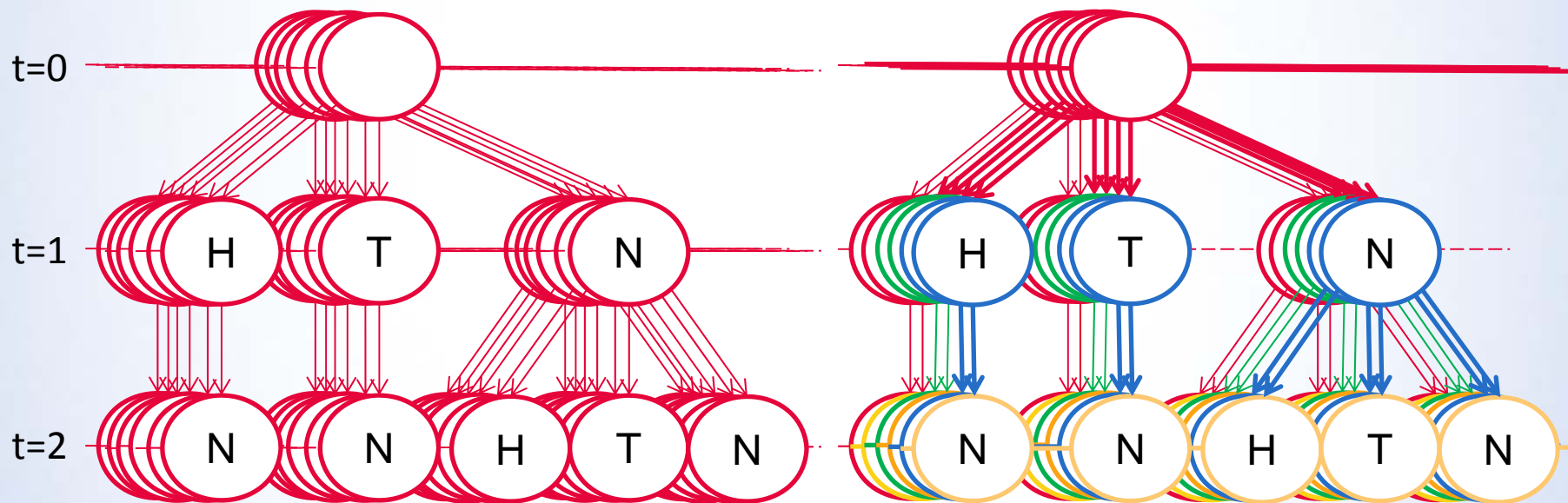
KEY CHALLENGES:

- This problem required a shift from
 - two-stage stochastic programming with simple recourse
 - to:
 - two-stage stochastic programming with recourse
- Requires a method of **resolving part of the uncertainty** while **maintaining** the **total uncertainty** of the problem.



WHAT IS THE DIFFERENCE?

- Stochastic programs can be approximated through a large number of scenarios. From LP to SP: H – harvest, T – Thin, N – Do nothing.



With simple recourse –
evaluated as penalties



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With recourse possibilities at $t=1$ and $t=2$.
Requires sorting of scenarios based on
remaining uncertainty

PROBLEM FORMULATION:

Objective function:

$$\max \sum_{n=1}^N p_n NPV_n - \lambda \sum_{t=1}^T \frac{CVaR_t}{T}$$

Subject to:

$$[1] NPV_n = \sum_{t=1}^T \frac{I_{nt} - w_t q}{(1+r)^{(t*5-u)}} + \sum_{j=1}^J \sum_k^{K_j} \frac{PV_{n6}}{(1+r)^{30}}, \forall n = 1, \dots, N$$

$$[2] L_{nt} = [I_{nt} - w_t q - b_t]^+, \forall n = 1, \dots, N, t = 1, \dots, T$$

$$[3] CVaR_t = \left(1 - \sum_{p=1}^t w_p\right) \left(Z_t + \frac{1}{(1-\alpha)N} \sum_{n=1}^N [L_{nt} - Z_t]^+\right)$$

$$+ \left(\sum_{p=1}^t w_p\right) \sum_{f=1}^F \frac{1}{F} \left(Z_t + \frac{1}{(1-\alpha)N_f^t} \sum_{n \in N_f^t} [L_{nt} - Z_t]^+\right), \forall t = 1, \dots, T$$

$$[4] \sum_{k=1}^{K_j} (x_{jkg} - x_{jkf}) s_{jkt} - \sum_{p=1}^t w_p * M \leq 0, f \in F, g \in F, j \in J, t = 2, \dots, T, f \neq g$$

$$[5] \sum_{k=1}^{K_j} (x_{jkf} - x_{jkg}) s_{jkt} - \sum_{p=1}^t w_p * M \leq 0, f \in F, g \in F, j \in J, t = 2, \dots, T, f \neq g$$

$$[6] \sum_{t=1}^T w_t \leq 1 \quad [7] I_{nt} = f(x_f) \quad [8] PV_{n6} = f(x_f) \quad [9] x_{jkg} \in \{0,1\}, w_t \in \{0,1\} \text{ for all } t \in T, f \in F, j \in J, k \in K$$

I - Income

PV - Productive value

x - decision

L - Loss

q - cost of inventory

b - periodic target

s - actions in schedule

M - Large number

λ - risk parameter

F - number of subsets

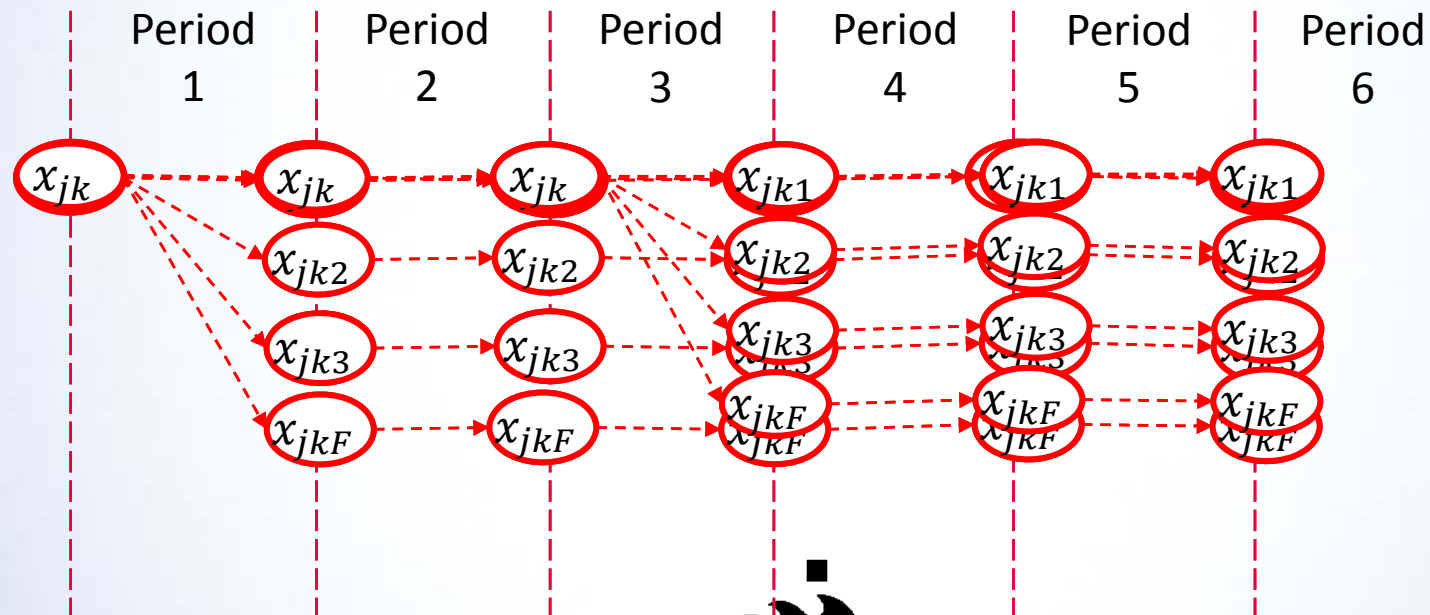


nonanticipativity
constraints

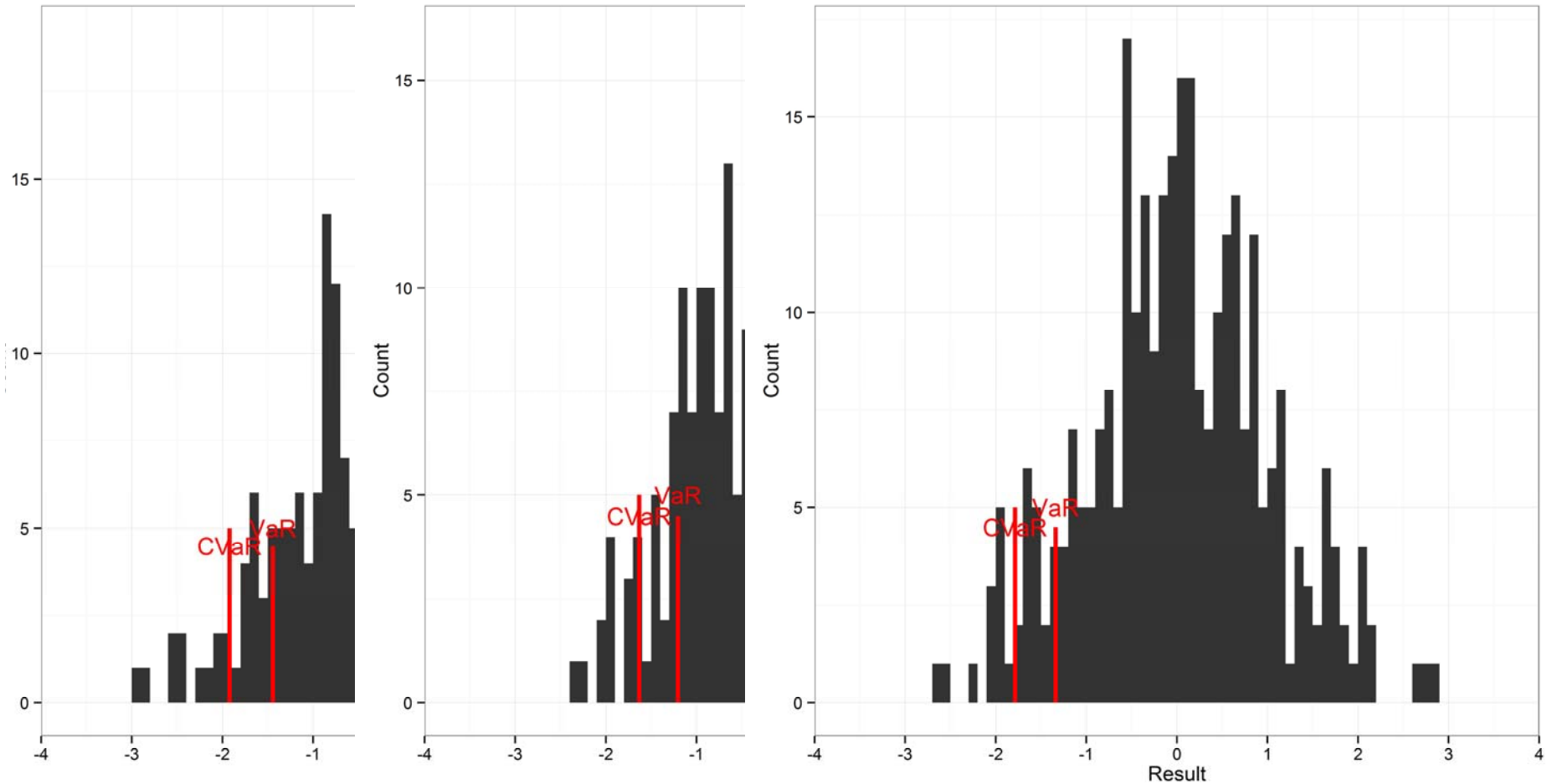


MANAGEMENT DECISIONS:

- Depending on the optimal timing will determine the shift from first stage to second stage.

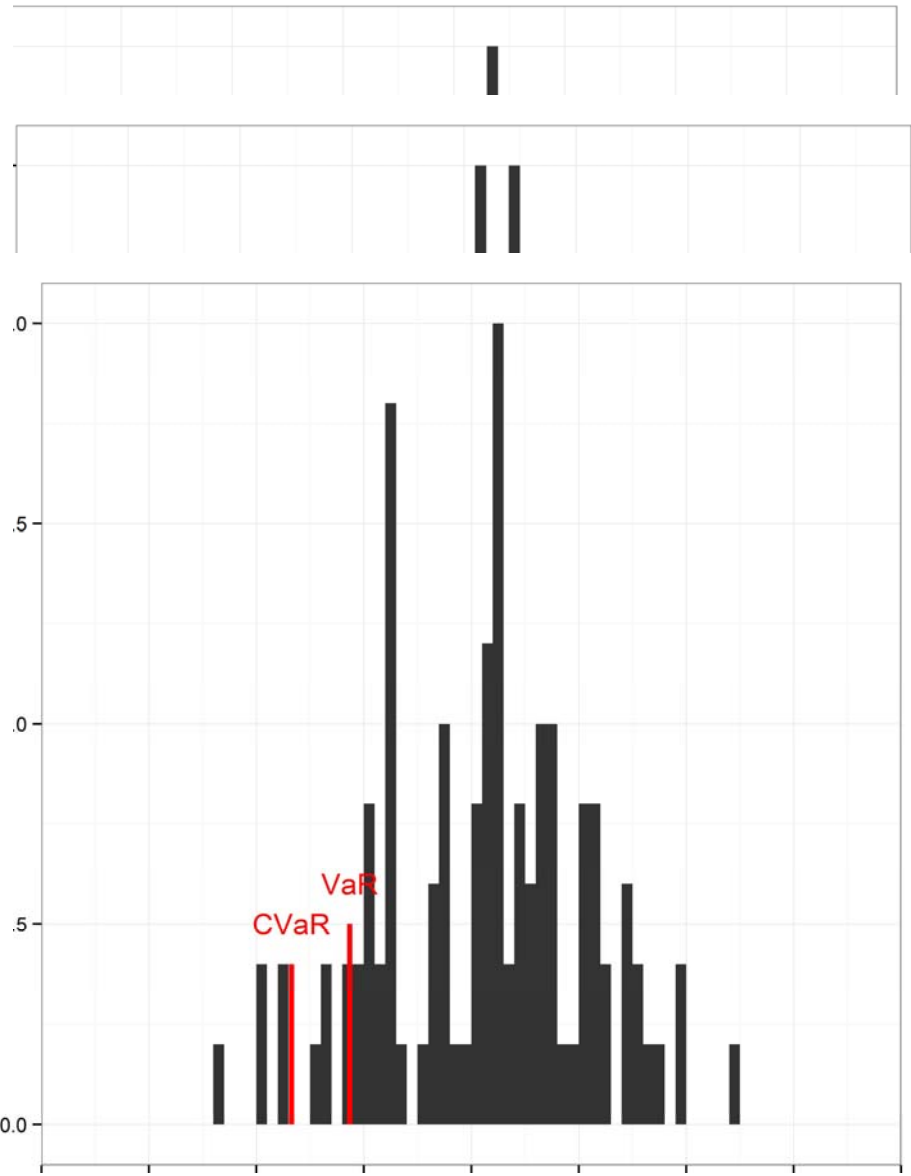
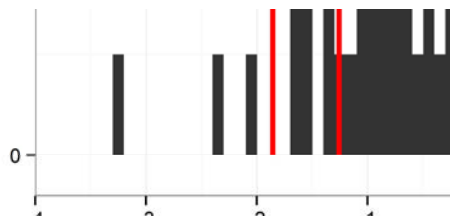
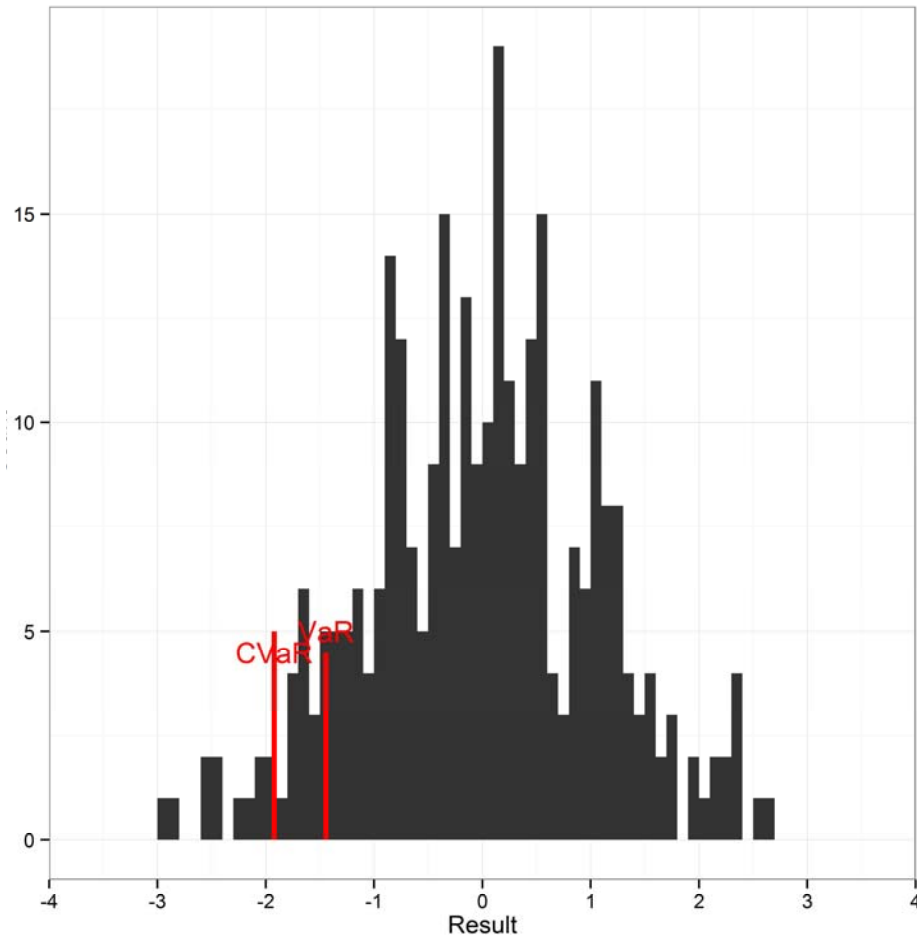


MANAGEMENT DECISIONS: CVaR ISSUES.



MANAGEMENT DECISIONS:

CVaR ISSUES

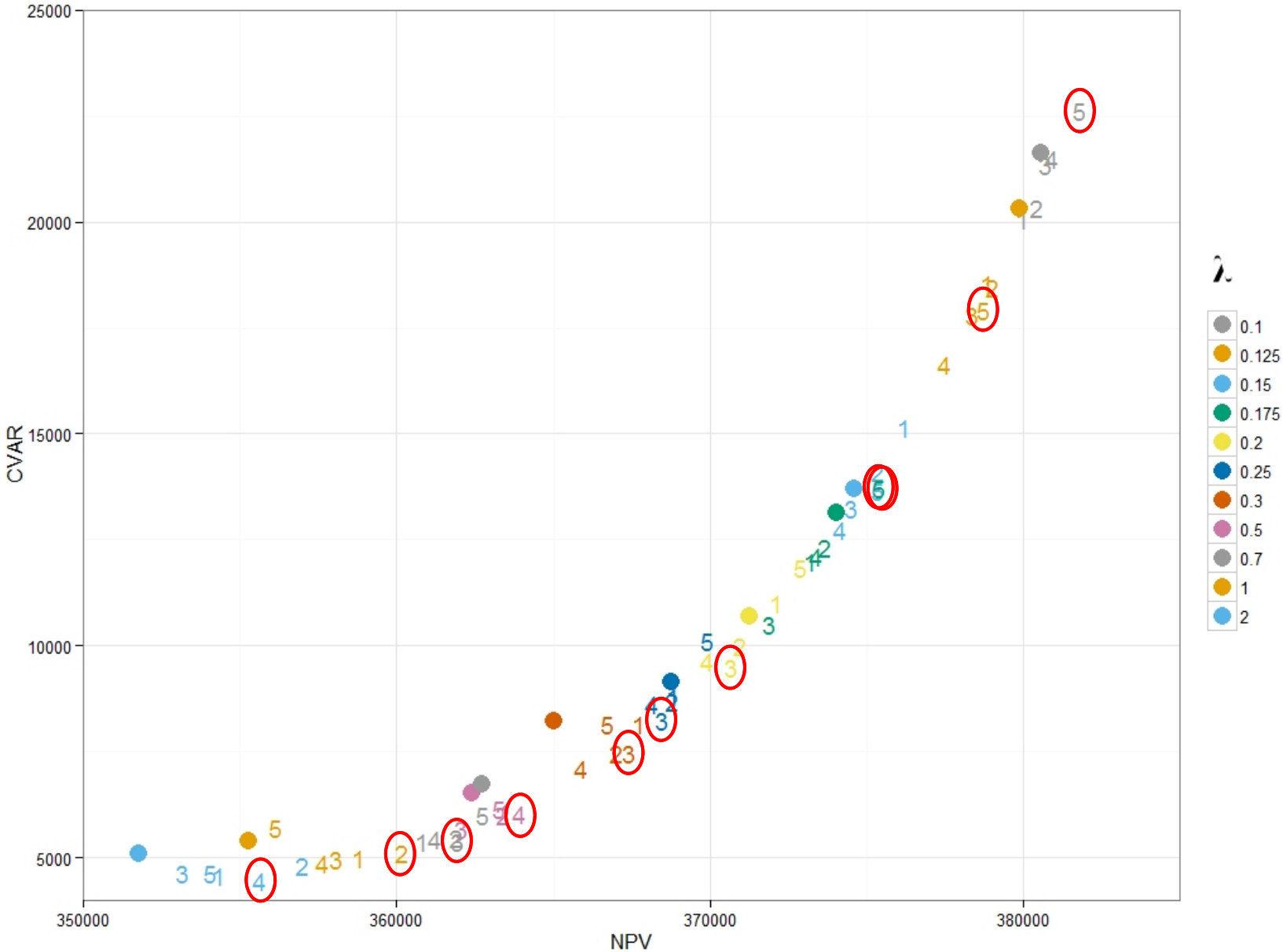


RESULTS:

- Four parameters of the model can be adjusted based on the preferences of the decision maker and current economic situation:
 - λ – risk parameter (varied from 0.1 – 2)
 - b_t - Periodic income target (60,000€/ period)
 - r – Interest rate (4%)
 - q – Inventory costs (500 €) – roughly 11€/ ha
- To highlight the shift, for each λ all possible options for conducting inventories were solved.



RESULTS:



CONCLUSIONS / MOVING FORWARD

- Demonstrated a two-stage stochastic programming with recourse
 - Allows for a resolution of uncertainty during the planning horizon.
- For this case:
 - With a near **risk neutral** DM ($\lambda = 0.1$ to 0.175), delay inventory until **end** of planning period.
 - With a **moderately risk averse** DM ($\lambda = 0.2$ to 0.3), delay inventory until **middle** of planning period
- Changing interest rate, costs of inventory will also cause a shift.



MOVING FORWARD

- Some assumptions made could be relaxed
 - (Which for a small holding may be true, but for a large holding may not...)
- Complete re-inventory → Partial re-inventory
 - select an area which benefits most from a new inventory
- Improvement in inventory method – reduction in uncertainty
 - future inventory methods may have improved accuracy



REFERENCES

- Rockafellar, R. T., and Uryasev, S. 2000. Optimization of conditional value-at-risk. *J. Risk*, **2**: 21-42.
- Pietilä, I., et al. (2010). Influence of growth prediction errors on the expected losses from forest decisions. *Silva Fennica* 44(5): 829-843.



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SORTING TO CREATE SECOND STAGE DATASETS:



- Large set of all scenarios:
 - N – red square
 - 1st opportunity to inventory
 - N_f^1 – blue, yellow, green rectangles
 - 2nd opportunity to inventory
 - N_f^2
 - Last opportunity to inventory
 - N_f^6
- N_f^t is a subset of N ,
- $N_{f_1}^t \cap N_{f_2}^t = \emptyset$ for all $f_1 \neq f_2$ and $\cup_{f \in F} N_f^t = N$

